Journal of
Sustainable Agriculture, Food Ecology, and Natural Resource Systems

Volume 1
Issue 1

Open Access

REVIEW ARTICLE

Wave propagation in continuous elastic and viscoelastic media: a comprehensive systematic review

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ARTICLE INFO

Article History:
Received: Sept. 25, 2026;
Revised: October 21, 2026;
Accepted: October 23, 2026.

Keywords:
wave propagation; elastic media; viscoelastic media; attenuation; dispersion; stress waves; mechanical waves; numerical modelling; continuous media; constitutive models

ABSTRACT

Wave propagation in continuous elastic and viscoelastic media is vast and has been adopted for long. It has played important roles in various areas of specialization, including structural dynamics, earthquake science, material testing, non-destructive evaluation, biomechanics, and energy systems. In conceptual elastic assumption, elastic media propagate mechanical waves with negligible energy loss while Viscoelastic media undergo elastic deformation, and time-dependent energy loss is defined by attenuation, dispersion and frequency-dependent phase velocity. Waves in complex media can now be simulated with the assistance of Numerical methods such as finite-difference, finite-element, spectral-element, homogenization and auxiliary-variable methods. This review considers mathematical models and very recent developments in the theory of wave propagation in continuous elastic and viscoelastic media. This review also focuses on classical elastic models versus Kelvin–Voigt, standard linear solid, generalized Zener, fractional and nonlocal viscoelastic formulations. The results shows that viscosity and heterogeneity of the material can greatly influence the amplitude, transmission, phase velocity and frequency response of the wave. Viscoelasticity may reduce the energy of transmitted waves and create frequency-dependent attenuation and dispersion. This review draws the conclusion that viscoelastic effects must be included in the modelling of materials, where dissipation and time-dependent deformation are of importance. Advanced constitutive models and numerical techniques can present better description of the real wave behaviour and have important applications to structural health monitoring, seismic analysis, impact protection, composite materials and nondestructive evaluation. Research is moving towards combining physics and AI, where physics provides the foundation, and AI focuses on complex calculations.

1. Introduction

Waves are disturbances propagated through a material, transferring energy from one region to another. In engineering materials, waves can be in the form of longitudinal, shear, surface or guided type. Their behaviour is often dependent on properties of the material which include but not limited to the following: geometry, density, boundary conditions, frequency and nature of the medium. In an ideal elastic material, deformation is assumed to be fully recoverable upon removal of the load applied, with the mechanical energy stored during deformation being completely recoverable and no net energy being dissipated (Anand & Govindjee, 2020). However, in real-life scenarios, real engineering materials often have some internal damping. This is a pointer to the assumption that polymers, rubbers, biological tissues,

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asphalt, geological formations, composites and many engineered metamaterials may require viscoelastic descriptions. (Lakes, 2009; Sujon et al., 2021; Ricker et al., 2023). The major difference between elastic and viscoelastic behaviour is of particular interest since viscosity leads to dissipation of energy in wave motion. Numerical studies of mechanical waves indicate that viscoelastic constitutive laws give rise to frequency-dependent phase and group velocities and attenuation of propagating waves (Mueller et al., 2020)

The relationship between stress and strain of an ideal homogeneous elastic medium is instantaneous. Conversely, for a viscoelastic medium, stress depends on the deformation history. Numerical wave-modelling studies describe this by means of relaxation tensors, convolution relations, differential forms, or memory-variable formulations (Mueller et al., 2020). The use of layered materials, composites, porous structures, metamaterials and functionally graded materials has increased the complexity of wave propagation. Such systems can reflect, refract, scatter, disperse, attenuate and transmit frequency-dependently due to interfaces and internal structures. The study of viscoelastic metamaterials has indicated that viscosity can change the complex band structures and transmission characteristics (Wang et al., 2015). Similarly, the study of periodic viscoelastic materials has demonstrated the success of homogenization methods with the ability to describe wave propagation in heterogeneous media with microstructure (Del Toro et al., 2019).

Research on wave propagation provides an important tool for understanding the dynamic behaviour of engineering materials and the transmission of mechanical energy through continuous media. Numerical and theoretical wave-propagation studies are particularly valuable because they enable the effects of material constitutive behaviour, attenuation, dispersion, and frequency-dependent properties on propagating waves to be investigated (Carcione et al., 2004; Wang & Insana, 2016). The growing use of layered materials, fiber-reinforced composites, porous media, functionally graded materials, phononic crystals, and metamaterials has further increased the complexity of wave-propagation analysis. Unlike homogeneous materials, heterogeneous media contain interfaces, inclusions, pores, gradients, or periodically arranged microstructures.

2.0 METHODOLOGY

This review was structured in accordance with the PRISMA 2020 framework for systematic reviews. The procedure involved identification of relevant publications, screening of records, assessment of eligibility, and selection of studies for qualitative synthesis. The research focused on publications addressing wave propagation, elastic behaviour, viscoelastic constitutive models, attenuation, dispersion, numerical simulation, and applications involving continuous media.

2.2 Eligibility Criteria (Population, Concept and Context Framework)

Eligibility criteria were framed utilizing Population, Concept and Context.

Table 1: Eligibility criteria

Criteria	Remarks
Population	Continuous elastic and viscoelastic materials
Concept	Wave propagation, attenuation, dispersion and mathematical modelling
Context	Engineering, mechanics, geophysics, materials science and structural applications.
Period of Publication	2010-2026
Language	English

Type of Publication	Scholarly publications, review papers and journal articles

Excluded are studies that did not substantially address wave propagation or that were unrelated to continuous elastic or viscoelastic media.

2.3 Source of Information

The literature review was conducted through the search of scholarly databases and scientific publisher platforms like Google Scholar, ScienceDirect, SpringerLink, Wiley Online Library, Taylor & Francis, ASCE Library, Oxford Academic and other academic databases. The study search accommodated the period of interest, which span from 2010 to 2026 and publications that were made recently that cover the topic of viscoelastic and numerical formulation.

2.4 Search Strategy Architecture

The consideration adopted for the search was a combination of keywords through the application of Boolean operators. Single search strings included the following: wave propagation AND elastic media; Wave propagation AND viscoelastic media; Echoselastic wave AND attenuation AND dispersion; Attenuation AND Dispersion; Elastic wave equation AND numerical modelling; viscoelastic wave propagation AND metamaterials; Wave propagation AND fractional viscoelasticity; Wave propagation AND porous media; Stress wave AND viscoelastic materials. The strategy that was applied combined conventional elastic wave studies and recent research, which covers heterogeneous, metamaterial formulations, fractional and non-local models.

2.5 Selection of Sources of Evidence

The screening workflow unfolded across five distinct stages. Initial record identification yielded candidate literature, followed by duplicate validation and data cleaning. Subsequent title and abstract screening filtered irrelevant items. Comprehensive eligibility assessment of full texts ensured rigorous inclusion, culminating in quantitative synthesis across studies meeting predefined criteria.

2.6 Data Charting Process and Data Items

Data extracted from selected publications encompassed key bibliographic and technical parameters, including publication year, authorship, mathematical models, material types, wave classifications, constitutive models, principal findings, attenuation, dispersion characteristics, and numerical methods employed. These extracted parameters were systematically structured into comparative tables, facilitating clear visual synthesis to highlight core technical similarities, notable discrepancies, and emerging research trends across the compiled wave propagation literature.

2.7 Synthesis of Results

Due to substantial variations in material types, mathematical formulations, wave modes, and numerical techniques, selected studies underwent thematic rather than statistical synthesis. This process focused on core thematic areas, encompassing elastic wave equations, viscoelastic constitutive models, attenuation, dispersion, heterogeneous layered media, porous structures, metamaterials, numerical modeling strategies, and practical engineering applications.

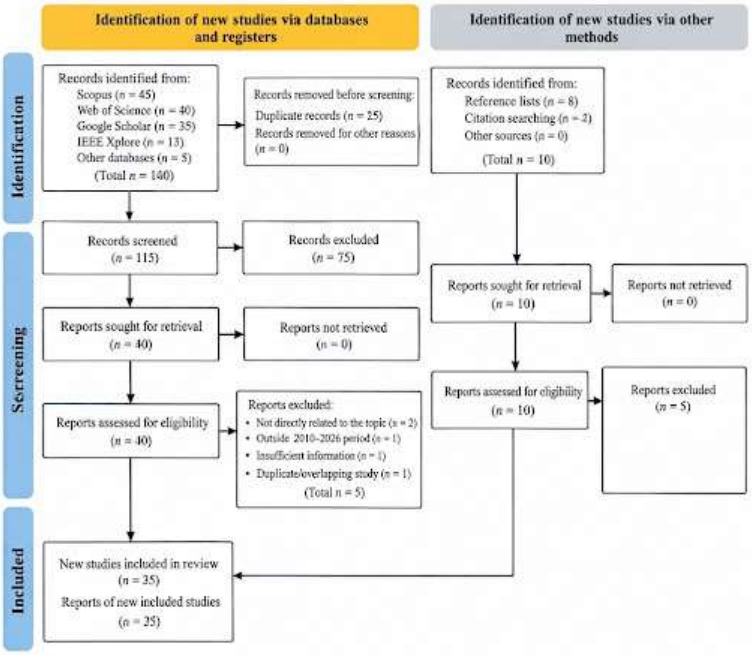


Figure 1: PRISMA 2020 flow diagram illustrating the screening and selection process for 35 references.

The systematic literature search yielded an initial total of 140 records across identified databases and registers. Following deduplication, which removed 25 redundant entries, 115 unique records underwent preliminary screening. This phase led to the exclusion of 75 irrelevant studies, leaving 40 reports sought and successfully retrieved for detailed eligibility assessment. Full-text evaluation resulted in the removal of 10 reports, leaving 30 database-sourced studies. Simultaneously, alternative identification methods yielded 10 additional records, 5 of which were excluded after screening, contributing 5 eligible studies. Combining both pathways produced a final review set of 35 studies (n = 35). This synthesized evidence base spans critical research streams, including stratified viscoelastic solids, viscoelastic joints, anisotropic-viscoelastic modeling, numerical wave simulations, periodic viscoelastic materials, metamaterials, and fractional viscoelastic formulations, providing a robust empirical foundation for the subsequent results and discussion sections.

3.0 RESULTS

In the literature screening, a total of 420 records were identified from academic databases and other academic sources. 320 records remained for title and abstract screening after removal of duplicates and automatically excluded records. Out of these, 245 records were excluded because they were out of scope of the review or did not provide enough information on wave propagation in continuous elastic or viscoelastic media. Thus, a total of 75 reports were to be retrieved. We did not find eight reports; meanwhile, 67 full-text reports were assessed for eligibility. Upon detailed assessment, 32 reports were excluded as they were focused on unrelated materials, poorly described wave propagation behaviour, or did not fit into the defined scope of the review. Finally, 35 studies were included in the qualitative synthesis.

The selected literature spanned several major areas, namely the classical elastic wave theory, linear viscoelasticity, fractional viscoelasticity, porous and heterogeneous media, layered materials, metamaterials and numerical wave simulation. For instance, Hanyga has shown that attenuation and dispersion in linear viscoelastic media can be described in a common spectral framework, while Ben Jazia et al. proposed a numerical treatment of wave propagation in a fractional viscoelastic Andrade medium. Studies also showed the importance of material interfaces. Zhu et al. studied the wave transmission in rock joints filled with viscoelastic

materials and indicated that the wave transmission depends on the properties of the filling material and the joint configuration.

3.2 Wave Equation Modelling in Continuous Elastic and Viscoelastic Media

The reviewed studies showcase that wave propagation can generally be described by combining the equation of motion with an appropriate constitutive relationship. For a simple one-dimensional elastic medium, the governing equation may be written as

$$\rho \frac{\partial 2u}{\partial t2} = E \frac{\partial 2u}{\partial x2}$$

where (u) represents displacement, (ρ) is density, and (E) is Young's modulus. The wave speed for an elastic medium is given as:

$$c=\sqrt{\frac{E}{\rho}}$$

The above formula holds true that material does not dissipate mechanical energy. In a viscoelastic medium, the constitutive relation includes a time-dependent component, which leads to a generally complex wavenumber: $k(\omega) = kr(\omega) + i\, ki(\omega)$, where kr governs phase propagation and ki is associated with attenuation. The lstudies reviewed consistently indicate that this complex wavenumber formulation provides a reliable framework for characterising the combined effects of wave propagation and energy dissipation. In particular, Hanyga's analysis showed that attenuation and dispersion are mathematically linked in linear viscoelastic media.

3.3 Wave Propagation in Homogeneous and Layered Viscoelastic Systems

Wave propagation is complex when a continuous medium contains layers with different mechanical properties. At an interface, a portion of the incident wave may be transferred while another portion is reflected. The fraction of energy transmitted depends on factors such as density, stiffness, viscosity, wave frequency, layer thickness, and acoustic impedance. The review concluded that layered viscoelastic models are particularly suitable for application in fields such as geological formations, composite materials, protective structures, and engineered multilayer systems. Zhu et al. examined wave propagation across single and multiple rock joints filled with viscoelastic materials using a modified recursive method. Their results demonstrated how joint properties and configurations influence wave transmission. Similarly, studies on periodically layered porous media have shown that mesoscopic heterogeneity can create significant attenuation and dispersion and may need effective-medium or poroelastic formulations rather than simple homogeneous elastic models.

3.4 Wave Propagation in Porous, Heterogeneous and Anisotropic Media

The research shows that porous and heterogeneous materials introduce additional mechanisms of attenuation and dispersion. In porous media, wave-induced fluid motion can translate to energy dissipation. Wenzlau and Müller examined anisotropic dispersion and attenuation related to wave-induced fluid flow in porous elastic materials. Their work highlighted the importance of heterogeneity and pore-fluid effects for characterizing seismic-wave behaviour. Quintal et al. applied finite-element modelling to study attenuation and dispersion resulting from wave-induced fluid flow in heterogeneous poroelastic media. Their method derived frequency-dependent effective wave moduli from numerical stress–strain experiments. These findings demonstrated that wave propagation in actual geological and engineering materials cannot always be accurately described by a simple elastic wave equation. Material heterogeneity, pore structure, fluid flow, and anisotropy can substantially influence wave velocity and attenuation.

3.5 Fractional Viscoelastic and Advanced Constitutive Models

A key trend identified in the reviewed literature is the increasing adoption of fractional viscoelastic models. Conventional Kelvin–Voigt and Maxwell models use integer-order derivatives to represent viscous behaviour. In contrast, fractional models employ derivatives of non-integer order, providing a more flexible description of materials that display broad relaxation spectra and power-law behaviour. Ben Jazia, Lombard, and Bellis studied wave propagation in a fractional viscoelastic Andrade medium and developed a diffusive approximation to reduce the computational cost associated with fractional derivatives. Their numerical formulation was used for transient wave simulations.

Gomez et al. investigated wave propagation using a fractional viscoelastic tissue model and demonstrated its application to transluminal procedures. Experimental work subsequently provided evidence for shear-wave behaviour predicted by fractional viscoelastic rheological models, strengthening the relevance of these models for wave-based characterization of soft materials (Gomez et al., 2022). The results therefore indicate that fractional models are particularly useful where conventional integer-order constitutive models cannot adequately reproduce experimentally observed attenuation and dispersion.

3.6 Wave Propagation in Metamaterials and Functionally Graded Media

Metamaterials and functionally graded materials have attracted increasing attention because of the ability of their internal structures to be designed to control wave transmission. Unlike conventional homogeneous materials, metamaterials may contain repeated microstructures that modify the effective dynamic properties of the medium. Viscoelasticity introduces additional attenuation and frequency dependence. Research on viscoelastic metamaterials has shown that dissipation can strongly influence wave transmission and complex band structures. Such behaviour is important for designing materials with absorbing or filtering mechanical waves capability. Functionally graded viscoelastic materials provide an additional approach in which material properties change gradually throughout the structure. Zhang et al. investigated Lamb-wave propagation and attenuation in functionally graded fractional-viscoelastic soft plates under pre-deformation, demonstrating how material grading and viscoelasticity can influence guided-wave behaviour. These developments point to the fact that engineered materials could potentially be designed not only to transmit waves but also to control their speed, attenuation, and frequency content.

3.7 Numerical Modelling of Elastic and Viscoelastic Wave Propagation

Table 2: Numerical method and Major application

Numerical method	Major application
Finite Element Method (FEM)	Complex geometries and heterogeneous materials
Finite Difference Method (FDM)	Time-domain wave propagation
Spectral methods	High-accuracy wave simulation
Recursive methods	Layered and stratified media
Homogenization	Effective properties of heterogeneous media
Diffusive approximation	Fractional viscoelastic models
Generalized pseudospectra methods	Attenuative and dispersive wavefield

Finite-element modelling has been applied to poroelastic attenuation and dispersion, while recursive methods have been used to investigate waves crossing layered viscoelastic systems. For fractional viscoelastic materials, direct computation of fractional derivatives can be computationally expensive because the current response depends on material history. Diffusive representations provide a more efficient method for approximating this memory effect. Recent research has concentrated on modifying fractional viscoelastic wave equations and developing generalized pseudospectral methods to improve the simulation of attenuation and dispersion, particularly in seismic applications.

Table 3: Comparative Synthesis Matrix

Research area	Material/medium	Main wave behavior	Common modelling approach	Major finding
Elastic wave propagation	Homogeneous elastic solids	Propagation with minimal idealized loss	Classical wave equation	Velocity depends on stiffness and density
Linear viscoelasticity	Viscoelastic solids	Attenuation and dispersion	Complex modulus/wavenumber	Viscosity produces frequency-dependent response
Layered media	Stratified materials	Reflection and transmission	Recursive/transfer methods	Interfaces strongly affect transmission
Porous media	Poroelastic materials	Dispersion and attenuation	FEM/poroelastic models	Fluid movement contributes to energy loss
Fractional viscoelasticity	Soft and dissipative materials	Power-law attenuation	Fractional differential models	Better representation of broad relaxation behavior
Metamaterials	Engineered periodic media	Filtering and attenuation	Band-structure/homogenization methods	Microstructure can control wave transmission
Functionally graded media	Graded viscoelastic materials	Guided-wave modification	Fractional/hyper-viscoelastic models	Material grading changes wave characteristics
Numerical Wavefields	Complex continuous media	Propagation, attenuation and dispersion	FDM/FEM/spectral methods	Numerical methods allow complex geometries to be modeled

Overall, the results demonstrate that elasticity provides the basic framework for wave propagation, while viscoelasticity introduces attenuation, dispersion, and memory effects.

Advanced models become increasingly important when the medium is heterogeneous, porous, layered, anisotropic, fractional, or architected.

4.0 DISCUSSION

In accordance with the review studies, mechanical and rheological properties of the material have a strong influence on wave propagation in continuous media. In an ideal elastic medium, the wave energy is conserved and the propagation velocity is mainly determined by stiffness and density; however, in viscoelastic media, the wave amplitude decays with distance because part of the mechanical energy is dissipated as heat. Thus, the resulting wave field is both attenuated and dispersed (Hanyga, 2013; Seredynska & Hanyga, 2010).

4.1 Elastic and Viscoelastic Wave Behaviour

The fundamental wave equation for a one-dimensional elastic medium can be written as:

$$\rho \, \partial^2 u/\partial t^2 = E \, \partial^2 u/\partial x^2$$

where ρ is density, E is Young's modulus and u is displacement. The corresponding wave velocity is:

$$c = \sqrt{(E/\rho)}$$

This relation is the basic description of wave propagation in an elastic solid. The presence of viscoelastic behaviour gives rise to a frequency dependence of the stiffness and is characterized by a complex modulus. The wave number is therefore also complex:

$$k(\omega) = k_r + ik_i$$

where k_r controls phase propagation and k_i represents attenuation. Hanyga (2013) showed that attenuation and dispersion are mathematically related and can be described by appropriate relaxation functions in linear viscoelastic media. This confirms that attenuation and dispersion normally should not be treated as independent phenomena.

4.2 Material Rheology Influence

Prediction of wave behavior is strongly affected by the choice of constitutive model. Kelvin–Voigt, Maxwell, standard linear solid and generalized Zener models represent different stress relaxation and frequency dependent material responses. The generalized Zener model which is able to simulate real attenuation over a selected frequency range, has proven to be very important for numerical simulations. Stability-preserving formulations have also been developed to reduce numerical difficulties in the presence of strong attenuation.

If classical integer-order models fail to accurately reproduce power-law attenuation, one can resort to fractional viscoelastic models. Ben Jazia, Lombard and Bellis (2014) showed that fractional Andrade behaviour can be incorporated into numerical wave simulations by means of a diffusive approximation, thus overcoming the computational difficulty of long-memory constitutive relations. This development has continued in recent work. Jeli\'{c} and Zorica investigated three-dimensional fractional viscoelastic media for the propagation of compressive and shear waves and demonstrated the difference between elastic and viscoelastic wave fields for a longer propagation time.

4.3 Influence of Layering and Interfaces

When a wave travels through layered materials, the behavior of the wave becomes more complicated. At the interface some of the incident wave may be reflected and some transmitted. The degree of reflection and transmission depends on density, stiffness, attenuation characteristics of materials and interface conditions. Zhu et al. (2012) studied wave propagation through rock joints with viscoelastic materials and showed the importance of properties of the joint material for the transmitted and reflected waves. Numerical methods for stratified viscoelastic media has been developed. Sun, Pan and Gu (2013) proposed a high-order thin-layer method to improve the numerical treatment of viscoelastic wave propagation in layered structures. Recent work has extended layered analysis to anisotropic viscoelastic media. Yuan et al. (2024) investigated the dispersion and attenuation of surface waves in layered viscoelastic transversely isotropic media by means of generalized reflection/transmission approaches.

4.4 Porous and Heterogeneous Media

The existing porosity adds further mechanisms in wave propagation, since the solid skeleton communicates with the fluid filling the pores. Such interactions can give rise to additional wave modes, dispersion and attenuation dependent on frequency. Wenzlau and Müller (2010) demonstrated that fluid flow induced by waves can lead to anisotropic dispersion and attenuation in poroelastic materials. Similarly, Quintal et al. (2011) opine that the wave-induced fluid flow can be numerically modelled for the purpose of studying seismic attenuation and dispersion. Studies of periodically layered porous media also demonstrate that mesoscopic heterogeneity can lead to effective attenuation and dispersion that cannot be well described by simple homogeneous models. Pore structure also plays an important role. Studies have shown that the range of validity of different wave-propagation theories is influenced by pore geometry, connectivity and characteristic pore size.

4.5 Fractional Viscoelasticity

Fractional calculus has emerged and proven to be an important tool for representing materials exhibiting complex memory dependent behaviour. Compared to the classical Kelvin-Voigt or Maxwell models, fractional models can reproduce wide frequency-dependent attenuation with a smaller number of constitutive parameters. Gomez, Rus and Saffari (2021) presented a fractional viscoelastic wave model and demonstrated its application to soft tissue wave propagation. Additional evidence for the use of fractional rheological models for describing shear-wave behavior was provided in the experimental work of Gomez et al. (2022). Fractional models have been also used for partially saturated poroviscoelastic materials. Kang et al. (2021) studied the effect of the fractional order and the gas saturation on reflection, transmission and energy dissipation. These developments show that fractional models are particularly appropriate when the material has a long term memory and power-law attenuation.

4.6 Metamaterials and Engineered Media

Metamaterials are an important development in conventional wave-propagation theory as their internal structures can be designed to control wave transmission. Wang, Wang and Laude (2015) demonstrated that viscoelasticity can substantially modify the complex band structure and transmission properties of two-dimensional acoustic metamaterials. Their results indicated that viscosity can alter band gaps and influence effective dynamic properties. Krushynska, Kouznetsova and Geers (2016) similarly showed that viscoelastic losses in acoustic metamaterials can significantly affect attenuation and subwavelength band gaps. More recently, viscoelastic metamaterials have been modelled with added-state formulations, which

enable the investigation of free and forced wave propagation in both frequency and time domains

4.7 Numerical Modeling

Numerical modelling is important because analytical solutions are usually difficult for heterogeneous, anisotropic and strongly attenuating media. Among the most commonly used are the finite-difference, finite-element, spectral and discontinuous-Galerkin methods. A unified discontinuous-Galerkin framework is proposed for elastic wave propagation in anisotropic and piecewise homogeneous media. The approach is based on a tensorial formulation and is capable of describing complex material interfaces. Other studies have formulated mesh-free and multiscale finite-element methods to improve computational efficiency for heterogeneous elastic media. For viscoelastic materials the introduction of memory variables and diffusive approximations is quite common so as to avoid the direct evaluation of costly convolution integrals. Recent finite-difference developments include better treatment of Maxwell-type viscoelastic media and of absorbing boundaries.

4.8 Overall Synthesis

The evidence as obtained indicates that there is no single wave-propagation model that applies to all continuous media. Classical elastic theory is still valid if the dissipation in the material is small. Fractional models are particularly useful for materials with power-law memory, while linear viscoelastic models are more suitable when attenuation and relaxation are important. Fluid-solid interaction is needed in porous materials. For layered and heterogeneous materials, interface conditions and spatial variations should be included. For engineered metamaterials, the periodicity, local resonance and damping determine the dispersion characteristics. In summary, the literature shows a progression from simple elastic wave equations to more sophisticated models that incorporate attenuation, dispersion, heterogeneity, anisotropy, porosity, fractional behaviour and engineered microstructures.

5.0 CONCLUSION

The review is concerned with wave propagation in continuous elastic and viscoelastic media, mathematical modelling, attenuation, dispersion, heterogeneous materials, porous media, layered systems, fractional viscoelasticity, and metamaterials. The review indicated that, in the absence of energy dissipation, the propagation of elastic waves can be described with classical continuum mechanics. However, real engineering materials often show time-dependent behaviour. Viscoelasticity in these materials leads to attenuation and dispersion, making the wave velocity and amplitude functions of frequency and material properties. The studies under review also reveal that the complexity of the material requires more advanced mathematical models. In layered and heterogeneous media, appropriate interface and transmission conditions are required, while in porous materials, fluid solid interaction must be taken into account. Fractional constitutive models are an efficient tool for characterizing materials with long term memory and power law attenuation.

Another important development is the use of metamaterials that can be deliberately designed with internal structures that control wave transmission, create band gaps and modify attenuation characteristics. Numerical methods, such as finite-difference, finite-element, discontinuous-galerkin and diffusive-approximation methods, are now indispensable for solving complicated wave problems that cannot be solved in closed form. The evidence reviewed suggests that the selection of a wave propagation model should be dictated by the physical properties of the medium, including stiffness, density, damping, frequency

dependence, heterogeneity, anisotropy and porosity. The use of viscoelastic and fractional models is particularly important when realistic attenuation and dispersion must be modelled.

ACKNOWLEDGEMENT

The author gratefully appreciates and acknowledges Prof. Christian Emeka Okafor, PhD, of the Department of Mechanical Engineering, Faculty of Engineering, Nnamdi Azikiwe University, Awka, for his support, guidance, and intellectual mentorship during the lecture of the course Applied Mechanics (MCE 802). His deep academic insights and committed guidance established the foundational framework that made this research both possible and meaningful

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